Ginzburg-Landau vortex lattice in bulk and film superconductors

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The Ginzburg-Landau (GL) theory [1] of 1950 is a very powerful phenomenological description of the thermodynamic and electrodynamic behavior of superconductors. In particular, the GL theory divides the superconducting materials into two groups: Type I superconductors (with GL parameter $\kappa < 0.707$) have positive energy of the wall separating normal conducting and superconducting domains, and type II superconductors (with $\kappa > 0.707$) have negative wall energy and are thus unstable with respect to the formation of inhomogeneities. By solving the GL equations, Abrikosov [2] showed that magnetic flux can penetrate type II superconductors in form of a lattice of vortices. For these discoveries Alexei A. Abrikosov and Vitalii L. Ginzburg were awarded the Nobel Prize in Physics in October 2003.

When a magnetic field H is applied parallel to a long type II superconductor, vortices start to penetrate when $\mu_0 H$ reaches the lower critical field $B_{c1} \approx \Phi_0(\ln\kappa + 0.5)/4\pi\lambda^2$, where $\Phi_0 = h/2e$ is the quantum of magnetic flux and λ the magnetic penetration depth. With increasing H the vortex density n and magnetic induction $B = n\Phi_0$ increase until the vortex cores overlap completely such that the bulk superconductvity vanishes at the upper critical field $\mu_0 H = B = B_{c2} = \Phi_0/2\pi\xi^2$, where $\xi = \lambda/\kappa$ is the GL coherence length. Abrikosov [2] obtained this qualitative picture by solving the linearized GL equations valid at large B near B_{c2} and the full GL equations at small $B \ll B_{c2}$ where the vortices are nearly isolated.

Approximate solutions of GL theory valid at all inductions where obtained by the circular cell method [3]. A very accurate and fast numerical method [4,5] uses a Fourier series ansatz for the periodic magnetic field B(x,y) and the order parameter $|\psi(x,y)|^2$ and obtains the Fourier coefficients by iteration. An example for these solutions is shown in Fig. 1 for the triangular vortex lattice with $\kappa = 5$ and vortex spacing $a = 4\lambda$ and 2λ, corresponding to $b = B/B_{c2} = 0.018$ and 0.072. The resulting magnetization curves in reduced units $h = \mu_0 H/B_{c2}$, $b = B/B_{c2}$, $m(b) = b - h(b)$ are shown in Fig. 2. It turns out that the old logarithmic approximation $-m = (1/4\kappa^2)\ln(0.36/b)$ applies only at very large $\kappa > 20$ in the small interval $1/2\kappa^2 < b < 0.01$ at very low (but not too low) inductions. Better fits for $\kappa > 20$ are given in [5]. For $\kappa < 20$ a very good fit to the exact h(b) which exactly satisfies the well known conditions $h(0) = h_{c1}$, $h'(0) = h''(0) = h''(1) = 0$, $h(1) = 1$, $h'(1) = 1 - p(\kappa)$ with $p(\kappa) = -dm/db|_{b=1} = 1/[(2\kappa^2 - 1)\beta_A + 1]$, $\beta_A = 1.15960$ (1.18034) for the triangular (square) vortex lattice, is (see dotted lines in Fig. 2):

$$h(b) = h_{c1} + c_1 b^3 / (1 + c_2 b + c_3 b^2)$$

with $h_{c1} = B_{c1}/B_{c2} = [\ln\kappa + \alpha(\kappa)] / 2\kappa^2$,

$$\alpha(\kappa) = 0.5 + (1 + \ln 2) / (2\kappa - 2^{1/2} + 2),$$

$$c_1 = (1 - h_{c1})^3 / (h_{c1} - p)^2,$$
$$c_2 = (1 - 3h_{c1} + 2p) / (h_{c1} - p),$$
$$c_3 = 1 + (1 - h_{c1})(1 - 2h_{c1} + p) / (h_{c1} - p)^2.$$

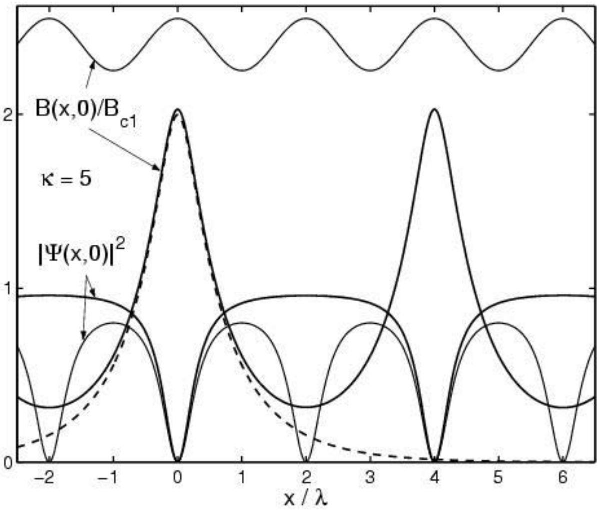
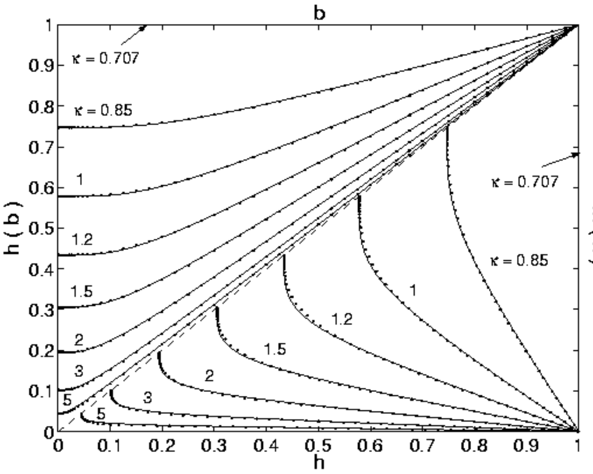


Fig. 1. Two computed profiles of the magnetic field B(x,y) and order parameter $|\psi(x,y)|^2$ of the triangular vortex lattice with spacings $a=4\lambda$ (solid lines) and $a=2\lambda$ (thin lines). The dashed lines show the magnetic field of an isolated vortex line.

Recently, the Fourier-series solution method [4,5] was extended [6] to the ideal periodic vortex lattice in infinite superconducting films of arbitrary thickness d put into a perpendicular magnetic field along z. Now $|\psi(x,y,z)|^2$ and $\mathbf{B}(x,y,z) = (B_x, B_y, B_z)$ depend on z. The GL free energy of a film contains also the energy of the magnetic stray field that is required to make $B_z(x,y,z)$ continuous across the surfaces $|z| = d/2$. As shown in Fig. 3, the magnetic field lines are now no longer parallel inside the superconductor but they widen near the surface to minimize the stray-field energy. For thick films with $d \gg \lambda$, the stray-field energy tends to a constant value. For thin films with $d \ll \lambda$, the modulation of **B**(x,y,z) is small when the vortex spacing is smaller than the effective penetration depth $\Lambda = \lambda^2/d$. This means that, as expected, the magnetic field in very thin films is nearly uniform and equal to the applied field. Only at very small applied fields with $a > \Lambda$, the vortex fields can be distinguished. The GL and London



solutions for isolated vortices, and vortices with well separated cores, in thick and thin flims are given in [7].

Fig. 2. Computed magnetization curves –m(h) and h(b) of the triangular GL vortex lattice (solid lines), coinciding within line thickness with those of the square vortex lattice. The dots show the above fit.

The above computations apply to ideal periodic vortex lattices for all values of b, κ, and d. For low b <<1 and not to small κ >>1, the properties of arbitray arrangements of straight or curved vortex lines can be calculated from London theory by linear superposition of the magnetic fields. For small displacements from the ideal triangular vortex lattice, approximate GL solutions can be written, from which the linear elastic energy of the vortex lattice can be obtained. Interestingly, it turns out that the elasticity of the vortex lattice is highly nonlocal [8], i.e., the elastic moduli for compression (c_{11}) and tilt (c_{44}) are dispersive, while the shear modulus c_{66} is approximately independent of the wave vector k of the strain. This elastic nonlocality makes the vortex lattice very soft for short-wavelength-distortions and increases both its thermal fluctuations and its pinning by material inhmogeneities [8]. One has approximately for not too small inductions $b > 1/2\kappa^2$:

$$c_{66} \approx (B\Phi_0 / 16\,\pi\,\mu_0\lambda^2)\,(1\text{-}b)^2 \ll c_{11}\,,$$
$$c_{11} \approx c_{44} \approx (B^2/\mu_0) / [\,1 + k^2\lambda^2/(1\text{-}b)\,]\,.$$

In High-T_c Superconductors (HTS), the vortex lattice is further softened by the strong anisotropy of the magnetic penetration depth. In highly anisotropic layered superconductors, the superconducting layers can be nearly decoupled, and the tilt modulus of the perpendicular vortices ("pancake stacks") may be even smaller than the shear modulus.

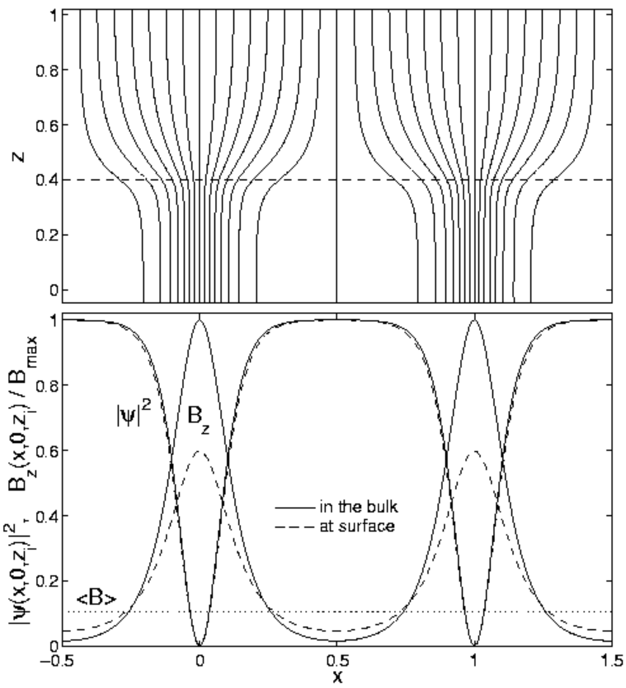


Fig. 3. The magnetic field lines (top) and profiles of the order parameter $|\psi(x,y,z)|^2$ and magnetic field $B_z(x,y,z)$ (bottom) for a superconducting thick film or half space, calculated from GL theory for induction b=0.04 and κ = 1.4, triangular vortex lattice with spacing a ≈10λ (a = unit length), film thickness d = 0.8a ≈ 8λ. Bottom: The solid lines show the profiles in the central plane z=0 (or in the bulk). The dashed lines show the profiles at the surfaces |z| = d/2. The dotted line marks the average induction B.

The perhaps most fascinating recent discovery [9] is that the "Josephson vortices" that run between the layers, can be decorated by "pancake vortices" that they attract and which indeed were observed [10] at the surface of layered HTS in form of dense vortex rows.

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