Composite fermions, trios and quartets in Fermi-Bose mixture.

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We consider a model of Fermi-Bose mixture for neutral particles in magnetic traps and for spinons and holons in high-T_c superconductors. For particles in magnetic traps we consider a case of short-range attraction between fermions and bosons and find the energies corresponding to the bound state of composite fermions fb, trios fbb and quartets $fbfb$. For high-T_c superconductors we consider a confinement interaction potential between spinons and holons, and find the energies corresponding to the bound states of composite hole $h = fb$ and for two holes $hh = fbfb$ configuration.

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I. INTRODUCTION.

The model of a Fermi-Bose mixture is very popular nowadays in connection with different problems in condensed matter physics such as high-T_c superconductivity, superfluidity in ^{3}He-^{4}He mixtures [1], fermionic superfluidity in magnetic traps and so on.

In high-T_c superconductivity this model was firstly proposed by J.Ranninger [2, 3] to describe simultaneously high transition temperature and short coherence length of SC pairs on one hand and the presence of well-defined Fermi-surface on the other. Later on P.W.Anderson [4] reformulated this model including bosonic degrees of freedom (holons b with spin 0 and charge e) and fermionic degrees of freedom (spinons f with charge 0 and spin 1/2). On the mean field level, according to the ideas of Anderson and Lee [5, 6], there is a spin-charge separation between spinons and holons. The interaction between them arises only on the level of fluctuations on top of the mean-field results. In our lecture we advocate a different philosophy which is more close to the ideas of Laughlin [7–9]. Namely, in accordance with Bulaevskii et al., it is a strong string-like interaction between spinons and holons [10, 11] which produces a spin-charge confinement inside a physical hole $h = fb$. The superconductive pair in this scenario is formed by the composite holes $\Delta = \langle h, h\rangle = \langle fb, fb\rangle$. According to this philosophy in all low temperature experiments in high-T_c superconductors the physical holes are seen as point-like structureless objects. Their composite nature (nontrivial formfactors) manifests itself only at high temperatures or high frequencies, where excited levels in the confinement potential become occupied. This ideology more naturally explains the results of APRES - photoemission [12] together with numerical simulations [13], which reveal rather standard Bogolubov type behavior of quasiparticles below T_c. In order to observe the composite nature of physical holes, we should study more carefully the optical conductivity $\sigma(\omega, T)$ and spin susceptibility $\chi(\omega, T)$ at frequencies and temperatures of the order of level spacing in the confinement potential $V(r) = \alpha(r/d)^\beta/2$, where $\alpha = zJS^2/2$, J is AFM exchange interaction, $z = 4$ is a number of nearest neighbors on a square lattcie in 2D, d is an intersite distance, $1 \le \beta \le 2$. Note that we consider holons to be the light particles with a mass $m_b \sim 1/t$, t is a hopping integral, and spinons to be heavy particles with a mass $m_f \sim 1/J$.

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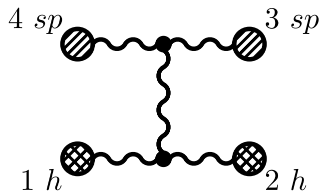

FIG. 1: The diagonalized Hamiltonian corresponds to three string-like potentials: between two holons (1 and 3), between two spinons (2 and 4) and between centers of masses of two spinons and two holons.

Fermi-Bose mixture of neutral particles

We start from Fermi-Bose mixture of neutral particles in a magnetic trap. Here we consider the Hamiltonian with a short-range attraction between fermions and bosons $U_{fb} < 0$. It leads to a bound state of fermion and boson on the 2D lattice with an energy [14]

$$E_b = \frac{1}{2m_{fb}a^2} = \frac{1}{2m_{fb}d^2}\frac{1}{\exp\left(2\pi/m_{fb}|U_{fb}|\right) - 1} \quad (1)$$

where $m_{fb} = m_b m_f/(m_b + m_f)$ is a reduced mass and a is a size of a composite fermion fb. If $E_b > T_{0b}, T_{0f}$, where T_{0f} and T_{0b} are degeneracy temperatures for fermions and bosons, then the creation of composite fermions fb takes place earlier (at higher temperatures) then Cooper pairing of two fermions $\langle ff \rangle$ or Bose-Einstein condensation of one or two bosons ($\langle b \rangle$ or$\langle bb \rangle$) [15]. The crossover temperature is governed by the Saha formula [16], which reads for equal densities of fermions and bosons ($n_b = n_f = n$): $T_* \sim E_b/\ln(E_b/T_{0fb})$, where $T_{0fb} = 2\pi n/m_{fb}$. Below T_* most of the particles in the mixture are paired in composite fermions fb. We calculate the residual interaction between two composite fermions and find that it corresponds to attraction. As a result the quartets $fbfb$ are formed. In the exchange approximation the bound state of a quartet for equal masses ($m_b = m_f$) of fermions and bosons has an energy $E_4 \approx 3E_b$ in a 2D case [14]. At low temperatures the quartets are dominant in the system and are bose-condensed below the temperature $T_c \approx T_0/8 \ln\ln(4/na^2)$. At intermediate temperatures some amount of trios fbb is also present in the system. Their binding energy in the exchange approximation is given by $E_3 \approx 1.7E_b$ for $m_b = m_f$ in 2D.

Strongly interacting Fermi-Bose mixture of spinons and holons

For the high-T_c case we consider a strongly interacting Fermi-Bose mixture of spinons and holons. We assume for simplicity that spinons and holons are interacting via harmonic oscillator potential $V(r) = \alpha(r/d)^2/2$ [17] and solve a four particle problem for two spinons and two holons. It has a Hamiltonian:

$$\hat{H}_{1234} = -\frac{\hbar^2}{2m_b}\Delta_1 - \frac{\hbar^2}{2m_f}\Delta_2 - \frac{\hbar^2}{2m_b}\Delta_3 - \frac{\hbar^2}{2m_f}\Delta_4 + V_{12} + V_{34} + V_{14} + V_{23} \quad (2)$$

where $m_b = 1/t$, $m_f = 1/J$, $V_{ij} = (zJS^2/2d^2)r_{ij}^2$; $\vec{r}_{ij} = \vec{r}_i - \vec{r}_j$. In Eq. (1) we neglect direct boson-boson and fermion-fermion interactions.

The direct diagonalization of the Hamiltonian yields the following expression for the energy of a bound state of four particles in a physical limit $J \ll t$:

$$E_4 \approx 2\sqrt{2}\omega_0 \left[1 + o\left(\frac{J}{t}\right)\right] + \frac{Q_{cm}^2}{2M_4} \quad (3)$$

where Q_{cm} is a center of mass momentum, $M_4 = 2m_b + 2m_f$ is a total mass of four particles, and

$$E_B \equiv \omega_0 = \sqrt{\frac{2\alpha}{m_{fb}d^2}} \approx \sqrt{\frac{2\alpha}{m_b d^2}} \sim \sqrt{Jt}$$

is a binding energy of a composite hole. Hence we get that $E_4 > 2E_B$ - two strings repel each other. It is interesting to note that the diagonalized Hamiltonian corresponds to three effective string-like interactions: between two spinons, between two holons, and between centers of masses of two spinons and two holons (see Fig. 1). Geometrically this solution resembles the situation in the two-leg ladders, where at a strong coupling along the rungs two holons are localized on one rung and two spinons are localized on another rung [18]. We are currently investigating the symmetry of the superconductive gap in our model.

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